UGC MODEL SYLLABUS

B.A./B.Sc. (HONOURS) PART-III : MATHEMATICS

(Duration : Two Semesters / One Year)

BMH 301 (a & b) ANALYSIS

Real Analysis

Riemann integral. Integrability of continuous and monotonic functions. The fundamental theorem of integral calculus. Mean value theorems of integral calculus.

Improper integrals and their convergence, Comparison tests, Abel's and Dirichlet's tests. Frullani's integral. Integral as a function of a parameter. Continuity, derivability and integrability of an integral of a function of a parameter.

Series of arbitrary terms. Convergence, divergence and Oscillation. Abel's and Dirichlet's tests. Multiplication of series. Double series.

Partial derivation and differentiability of *real-valued* functions of two variables. Schwarz and Young's theorem. Implicit function theorem.

Fourier series. Fourier expansion of piecewise monotonic functions.

Complex Analysis

Complex numbers as ordered pairs. Geometric representation of Complex numbers. Stereographic projection.

Continuity and differentiability of Complex functions. Analytic functions. Cauchy-Riemann equations. Harmonic functions.

Elementary functions. Mapping by elementary functions.

Mobius transformations. Fixed points. Cross ratio. Inverse points and critical mappings. Conformal mappings.

Metric Spaces

Definition and examples of metric spaces. Neighbourhoods. Limit points. Interior points. Open and closed sets. Closure and interior. Boundary points. Sub-space of a metric space. Cauchy sequences. Completeness. Cantor's intersection theorem. Contraction principle. Construction of real numbers as the completion of the incomplete metric space of rationals. Real numbers as a complete ordered field. Dense subsets. Baire Category theorem. Separable, second countable and first countable spaces. Continuous functions. Extension theorem. Uniform continuity. Isometry and homeomorphism. Equivalent metrics. Compactness. Sequential compactness. Totally bounded spaces. Finite intersection property. Continuous functions and compact sets. Connectedness. Components. Continuous functions and connected sets.

BMH 302 (a & b) ABSTRACT ALGEBRA

Group– Automorphisms, inner automorphism. Automorphism groups and their computations. Conjugacy relation. Normaliser. Counting principle and the class equation of a finite group. Center for Group of prime-order. Abelianizing of a group and its universal property. Sylow's theorems. p-Sylow subgroup. Structure theorem for finite Abelian groups.

Ring theory–Ring homomorphism. Ideals and Quotient Rings. Field of Quotients of an Integral Domain. Euclidean Rings. Polynomial Rings. Polynomials over the Rational Field. The Eisenstien Criterion. Polynomial Rings over Commutative Rings. Unique factorization domain. R unique factorisation domain implies so is $R[x_1, x_2 .....x_n]$.

Definition and examples of vector spaces. Subspaces. Sum and direct sum of subspaces. Linear span. Linear dependence, independence and their basic properties. Basis. Finite dimensional vector spaces. Existence theorem for bases. Invariance of the number of elements of a basis set. Dimension. Existence of complementary subspace of a subspace of a finite dimensional vector space. Dimension of sums of subspaces. Quotient space and its dimension. Linear transformations and their representation as matrices. The Algebra of linear transformations. The rank nullity theorem. Change of basis. Dual space. Bidual space and natural isomorphism. Adjoint of a linear transformation. Eigenvalues and eigenvectors of a linear transformation. Diagonalisation. Annihilator of a subspace. Bilinear, Quadratic and Hermitian forms.

Inner Product Spaces-Cauchy-Schwarz inequality. Orthogonal vectors. Orthogonal Complements. Orthonormal sets and bases. Bessel's inequality for finite dimensional spaces. Gram-Schmidt Orthogonalization process.

Modules, Submodules, Quotient modules, Homomorphism and Isomorphism theorems.

BMH 303 (a & b) PROGRAMMING IN C AND NUMERICAL ANALYSIS

Programming in C

Programmer's model of a computer. Algorithms. Flow Charts. Data Types. Arithmetic and input/output instructions. Decisions control structures. Decision statements. Logical and Conditional operators. Loop. Case control structures. Functions. Recursions. Preprocessors. Arrays. Puppetting of strings. Structures. Pointers. File formatting.

Numerical Analysis

Solution of Equations: Bisection, Secant, Regula Falsi, Newton's Method, Roots of Polynomials.

Interpolation: Lagrange and Hermite Interpolation, Divided Differences, Difference Schemes, Interpolation Formulas using Differences.

Numerical Differentiation.

Numerical Quadrature: Newton-Cote's Formulas, Gauss Quadrature Formulas, Chebychev's Formulas.

Linear Equations: Direct Methods for Solving Systems of Linear Equations (Gauss Elimination, LU Decomposition, Cholesky Decomposition), Iterative Methods (Jacobi, Gauss-Seidel, Relaxation Methods).

The Algebraic Eigenvalue problem: Jacobi's Method, Givens' Method, Householder's Method, Power Method, QR Method, Lanczos' Method.

Ordinary Differential Equations: Euler Method, Single-step Methods, Runge-Kutta's Method, Multi-step Methods, Milne-Simpson Method, Methods Based on Numerical Integration, Methods Based on Numerical Differentiation, Boundary Value Problems, Eigenvalue Problems.

Approximation: Different Types of Approximation, Least Square Polynomial Approximation, Polynomial Approximation using Orthogonal Polynomials, Approximation with Trigonometric Functions, Exponential Functions, Chebychev Polynomials, Rational Functions.

Monte Carlo Methods

Random number generation, congruential generators, statistical tests of pseudo-random numbers.

Random variate generation, inverse transform method, composition method, acceptance-rejection method, generation of exponential, normal variates, binomial and Poisson variates.

Monte Carlo integration, hit or miss Monte Carlo integration, Monte Carlo integration for improper integrals, error analysis for Monte Carlo integration.

BMH 304 (a & b) PROBABILITY THEORY AND OPTIMIZATION

Probability Theory

Notion of probability: Random experiment, sample space, axiom of probability, elementary properties of probability, equally likely outcome problems.

Random Variables: Concept, cumulative distribution function, discrete and continuous random variables, expectations, mean, variance. moment generating function.

Discrete random variable: Bernoulli random variable, binomial random variable, geometric random variable. Poisson random variable.

Continuous random variables: Uniform random variable, exponential random variable, Gamma random variable, normal random variable.

Conditional probability and conditional expectations, Bayes theorem. independence, computing expectation by conditioning; some applications – a list model, a random graph, Palya's urn model.

Bivariate random variables: Joint distribution, joint and conditional distributions, the correlation coefficient.

Functions of random variables: Sum of random variables, the law of large numbers and central limit theorem, the approximation of distributions.

Uncertainty, information and entropy, conditional entropy. solution of certain logical problems by calculating information.

Optimization

The linear programming problem. Problem formulation. Linear programming in matrix notation. Graphical solution of linear programming problems. Some basic properties of convex sets, convex functions and concave functions. Theory and application of the simplex method of solution of a linear programming problem. Charne's M-Technique. The two phase method. Principle of duality in linear programming problem. Fundamental duality theorem. Simple problems. The Transportation and Assignment problems.